Periodicity and Texel Size Detection Using Sum and Difference Histograms

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Abstract. Texture periodicity and texture element (texel) size are important characteristics for texture recognition and discrimination. In this paper, an approach to determine both, texture periodicity and texel size, is proposed. Our method is based on the entropy, a texture measure computed from the Sum and Difference Histograms. The entropy value is sensitive to histograms parameters and takes its lower value when the parameters match with texel size or its integer multiples, in an specific direction. We show the performance of our method by texture synthesis, tiling a sample of the detected size and measuring the similarity between the original image and the synthesized one, showing good results with regular textures and texels with different shapes.

Key words: Texel Size, Texture Periodicity, Entropy, Texture Analysis, Texture Synthesis.

1 Introduction

Texture analysis is an important issue in computer vision. Texture is a visual property perceived in all objects around us. There is not a formal definition of visual texture, but from the structural point of view, it is widely accepted to define it as an image conformed by two components: a texture element (texel), which is the fundamental micro-structure in generic natural images [14], and its placement rules. The main task of structural analysis is to identify texels and their placement rules, which can be related with texture periodicity. These components are sought in order to obtain a compact description of the analyzed texture, which could be used in applications of texture synthesis [2], high speed image transmission, texture compression, among others. In the same way, the texel can be used as reference in order to improve performance in classification [5][8] and segmentation [12] tasks and achieve invariant texture analysis [6]. In the case of natural textures it is not trivial to analyze periodicity since it is reduced by variations in elements, like illumination changes, perspective deformations, shape, and same stochastic nature of texture.

Coocurrence matrix (CM) proposed by Haralick [4] has been widely used to detect periodicity exploiting its parametrization, and mainly using two statistics: χ^2 and κ [11]. Other approaches have been proposed for the same task. Textural

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periodicity determination using a distance matching function has been proposed by Oh *et al.* [10], improving time consumption in comparison with inertias from the CM. Recently, Ahuja and Todorovic [1] have proposed the extraction of texels in 2.1D natural textures, using segmentation trees. Grigorescu and Petkov [3], estimate the minimum square window that corresponds with texel size of regular images based on the calculation of Renyi's generalized entropies. Leu [7] has proposed the use of the gradient field and an autocorrelation function to determine the periodicity of a given texture. Nang and Pang [9] have proposed the regularity analysis of texture with a regular bands (RB) method.

In this paper, texture periodicity and texel size are determinated using the property of entropy calculated from the Sum and Difference Histograms (SDH) proposed by Unser [13]. SDH has a free parameter, a relative displacement vector (DV) which consists in two components, horizontal and vertical. When DV is varied, an entropy function with respect to DV is obtained. When DV matches with the texel size or an integer multiple, entropy function reaches local minima, such behavior allows us to detect period and hence, the texel size. Moreover, SDH present computational advantages over the CM in complexity and memory consumption since SDH decrease in one order the CM memory storage. The paper is structured as follows: Section 2 describes the calculation of SDH. Section 3 describes the proposed method to extract the periodicity and texel size. In Section 4 the experimental setup is described and results for artificial and natural textures with different randomness levels are presented in Section 5. Conclusions and future work are given in Section 6.

2 Sum and Difference Histogram

To obtain the SDH, let us define a texture image $I(m,n)$ of $M \times N$ pixels size, and K gray levels $k = \{0, 1, \ldots, K-1\}$. Let us consider a pixel localized in coordinates (m,n) with intensity denoted as $I_{m,n}$ and a second pixel in a relative position with intensity $I_{m+d_m,n+d_n}$, where (d_m, d_n) is the relative displacement vector (DV). Sum and difference, associated with the relative DV (d_m, d_n), are defined as:

$$s_{m,n} = I_{m,n} + I_{m+d_m,n+d_n}, \tag{1}$$

$$d_{m,n} = I_{m,n} - I_{m+d_n,n+d_n}. \tag{2}$$

Sum histogram h_s and difference histogram h_d with displacement vector (d_m, d_n) over the image domain D, are defined as: $h_s(i) = Card\{(m,n) \in D, s_{m,n} = i\}$ and $h_d(j) = Card\{(m,n) \in D, d_{m,n} = j\}$. Normalized SDH are estimations of the sum and difference probability functions $P_s(i)$ and $P_d(j)$.

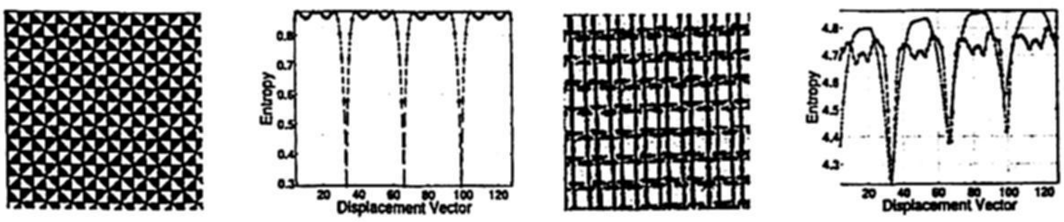
Different measures computed from the probability distributions have been proposed to be used as textural information. In this work, the measure used is the entropy, defined in Eq. 3.

$$\text{entropy} = -\sum_i P_s(i) \cdot log(P_s(i)) - \sum_j P_d(j) \cdot log(P_d(j)). \qquad (3)$$

3 Periodicity and Texel Size Determination

As was described previously, SDH are parameterized by a displacement vector which is represented in cartesian coordinates as (d_m, d_n). Let us take the periodic texture $I(m, n)$ previously described, with dimensions of $M \times N$ pixels and period T_m, T_n in horizontal and vertical directions, respectively. Using SDH we can obtain an entropy value $E(d_m, d_n)$ in function of the displacement vector $DV = d_m, d_n$. In a periodic texture, when DV matches the texture period (texel size), or multiples of it, entropy function reaches a minimum value. This behavior can be exploited in order to detect period for horizontal and vertical directions, when one of the parameters is fixed to zero. When $d_m = 0$, d_n takes values in the range $[2, \frac{N}{2}]$, and when $d_n = 0$, d_m is in the range of $[2, \frac{M}{2}]$. The upper value is proposed to assure at least two texels in the surface analyzed.

For example, an artificial and periodic texture with a square texel of 32×32 pixels size is presented in Fig. 1a. In Fig. 1b the entropy plots in both directions horizontal (with the marker (o)) and vertical (with the marker (+)) are given. We can see in this case, that the entropy function is the same in both directions, since this is a square texel. We also can see, that the entropy plot is periodic and has minima when DV matches with integer multiples of the texel size (e.g., 32, 64, 96,...).



(a) (b) (c) (d)

Fig. 1. An artificial texture pattern (a) and its entropy function in both directions (b). A natural texture pattern (c) and its entropy function in both directions (d).

We can determine the texel size of an artificial texture pattern by finding the first global minimum, but it is more difficult when a natural texture is analyzed. Changes in texel shape and its placement rules reduce periodicity due to the typical irregularities said before. Lets take the natural texture pattern shown in Fig. 1c, whom texel size is approximately 33×33 pixels size. By analyzing this texture pattern with the same criteria, we obtain the entropy functions shown in Fig. 1d. Although functions are not periodic, they still have local minima when DV corresponds with the texel size. Including natural cases in the general criteria, we can say that texel size is determined by the DV where the global minimum from entropy functions is found.

4 Experimental Setup

To test the performance of our approach, we use 9 images: synthetic textures, natural quasi-periodic textures and a natural stochastic texture. The image data set (see Fig. 2) consists of an artificial texture and 8 images extracted from the Brodatz album [?]. Take notice that although natural images are apparently regular, a detailed inspection shows that the position rules vary, considering them as quasi-periodic. Last image (D9), can be defined as stochastic since it looks like 2D noise.

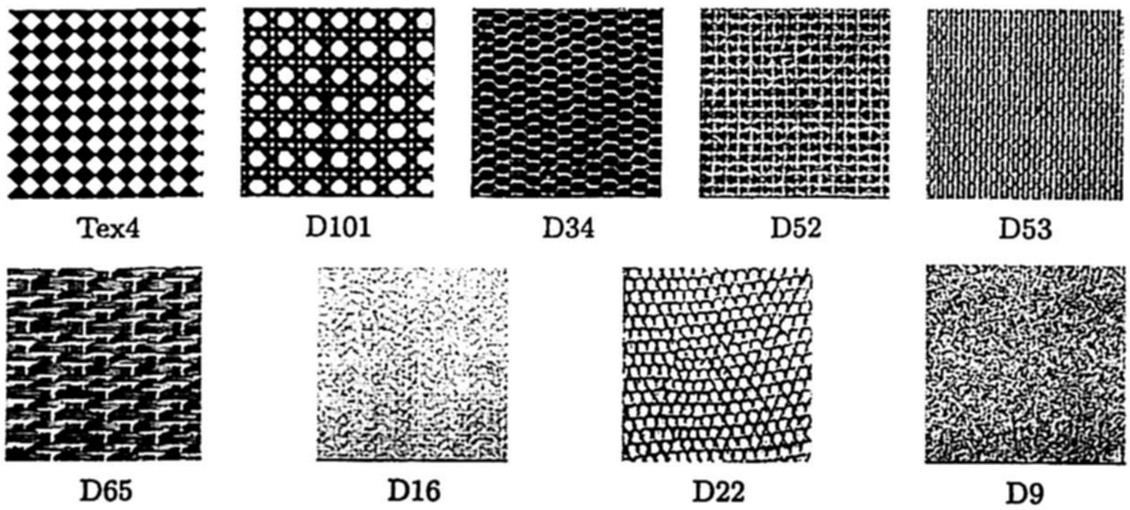


Fig. 2. Data Set used in experiments.

Performance is evaluated by taking a sample from the original image and using it in texture synthesis. Texture synthesis has been an important issue in computer vision, used as a way to verify texture analysis methods among other applications. The algorithm used in texture synthesis is tiling. Such algorithm consists in tiling an image with specific dimensions where the tile is a sample of a given texture. In this experiment, the tile is the sample taken from the original image. In order to quantify similarity between the original image and the synthetic image we use the Kolmogorov-Smirnov (KS) test defined in Eq. 4. KS test has values in the range [0,1] so we can have an intuitive result where zero is the value for two images that match exactly and one for two completely different images, like an image completely white and other completely black.

$$KS = max_k |H_1(k) - H_2(k)|, \tag{4}$$

where H_1 and H_2 are the cumulative distribution functions of two histograms.

5 Experimental Results

Entropy functions were calculated for each image, their plots are presented in Fig. 3, horizontal direction is presented with the mark (o) and vertical direction is presented with the mark (+). Table 1 shows the results of texel size detection, following the criterium that the texel size is found in the DV which corresponds to the global minimum.

In this image set, we can find four different cases. First case is the artificial texture named Tex4. This image is periodic in both directions, so entropy functions are periodic too. Since there are not a single global minimum, texel is detected in the first global minimum. In natural textures, there are specific cases, the first one is a quasi-periodic texture with square texel (e.g., D101). Its entropy functions show some periodicity in both directions and the global minimum in each direction matches with texel size. Another case is when there are periodicities in both directions but not with the same period, this means that the texel has a rectangular shape (e.g., D34, D52, D53, D65, D16). Last case within natural textures, is when there is not apparent regularity or it is difficult to describe a basic pattern and its position rules. This is the case of D9, where entropy plots looks like noise, the global minimums are localized in a high DV, hence the texel size tends to be big.

Table 1. Results in texel size determination and Kolmogorov-Smirnov test of synthesized textures and original textures.

Texture	Texel	KS Test	Texture	Texel	KS Test
Tex4	28×28	0.000	D65	28×69	0.110
D102	38×37	0.027	D16	7×43	0.044
D34	17×41	0.284	D22	32×15	0.110
D052	59×32	0.284	D9	83×109	0.052
D53	20×14	0.247			

In Fig. 4 the synthetic textures created from the texel detected are shown. The synthesis algorithm consists in taking the first sample of the size detected and repeating this sample in both horizontal and vertical direction through a specific surface. Results of KS test are shown in Table 1. These values show that the synthetic image is very similar to the original one with values lower than 0.3. We can see that the synthetic texture of the image named as Tex4, matches exactly with the original one obtaining a value of zero. Synthetic images

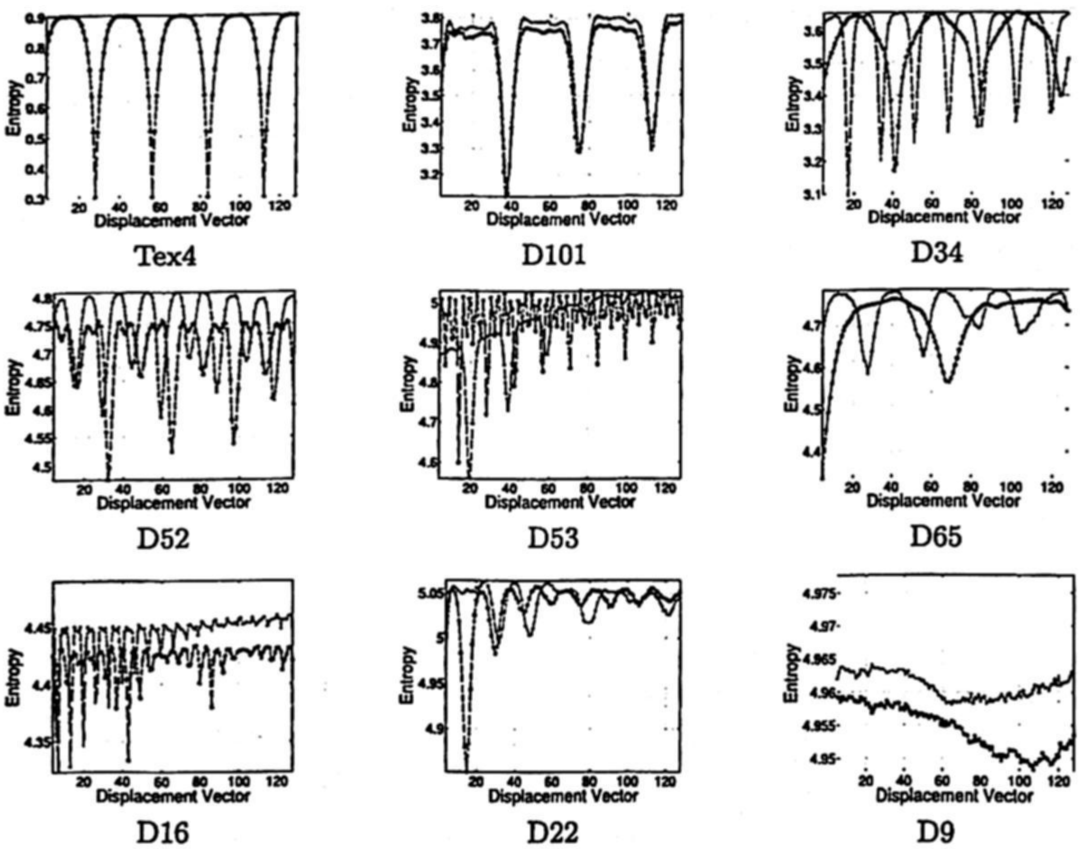


Fig. 3. Entropy plots for each image in horizontal (o) and vertical (+) directions.

corresponding with D101, D052, D16 and D9, exhibit good similarity values, lower than 0.1, this is, images are similar in more that 90%. The highest value of 0.28 is obtained with the image D34. In general, due to natural irregularities of texture, there are certain differences between the original and synthetic images. As tiling algorithm is regular, there is an error that accumulates and propagates through all the synthetic surface.

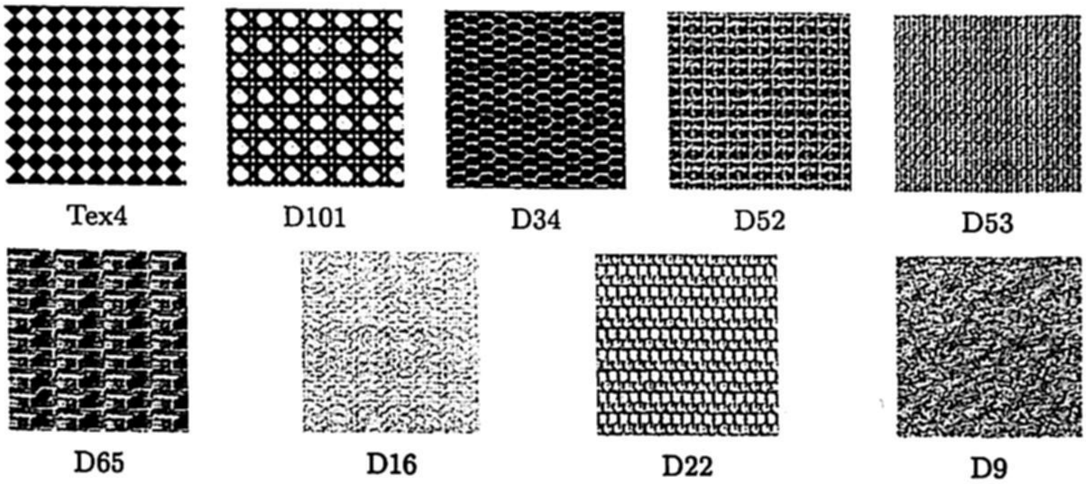


Fig. 4. **Synthesized textures by tiling detected texels. Compare with Fig. 2.**

6 Conclusion

Periodicity and texel size detection are classic problems in structural texture analysis. In this paper a method based on the property of entropy, calculated from the SDH, to detect the texel size was proposed. Entropy is calculated as a function of the DV in SDH. When DV matches texel size or its multiples, the entropy function has local minima, but texel size is found in the DV where the global minimum is found. Performance was evaluated qualitatively with texture synthesis by tiling, and quantitatively with the Kolmogorov-Smirnov test, showing good results with periodic, quasi-periodic and stochastic textures.

The main advantage of this method over the based on the CM is the use of the SDH that simplifies the computational complexity and memory storage. In an image with K grey levels, the CM requires $K \times K$ memory locations, while the SDH only requires $2K - 1$. Other advantage is the analysis in two directions by fixing to zero a component of the DV. Doing so, we can detect different shapes of texels, accurately identifying a squared or rectangular texel.

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